

# **Practical training**

# AN INTRODUCTION TO ECONOMIC OPTIMIZATION:

# MS EXCEL SOLVER TOOL

**Topic:** *Basic operations with financial functions. Study of formulas that calculate investment and depreciation of assets.* 

**Goal:** To master the following techniques: 1. Understanding how Excel rounds up values. 2. Using basic investment functions. 3. Using basic depreciation functions.

Microsoft Excel Solver is an optimisation tool, inbuilt provided into Excel. This tool can be used to solve practical business problems like optimum product mix problems, transportation or distribution problems, capital Budgeting problems, retirement planning problems, and work-force scheduling problems.

# What are Solvers Good For?

Solvers, or optimizers, are software tools that help users determine the best way to do something. The "something" might involve allocating money to investments, or locating new warehouse facilities, or scheduling hospital operating rooms. In each case, multiple decisions need to be made in the best possible way while simultaneously satisfying a number of logical conditions (or constraints). The "best" or optimal solution may mean maximizing profits, minimizing costs, or achieving the best possible quality. Here are some representative examples of optimization problems:

# 1. Finance and Investment

**Working capital management** involves allocating cash to different purposes (accounts receivable, inventory, etc.) across multiple time periods, to maximize interest earnings.

**Capital budgeting** involves allocating funds to projects that initially consume cash but later generate cash, to maximize a firm's return on capital.

**Portfolio optimization** -- creating "efficient portfolios" -- involves allocating funds to stocks or bonds to maximize return for a given level of risk, or to minimize risk for a target rate of return.

# 2. Manufacturing

**Job shop scheduling** involves allocating time for work orders on different types of production equipment, to minimize delivery time or maximize equipment utilization.

**Blending** (of petroleum products, ores, animal feed, etc.) involves allocating and combining raw materials of different types and grades, to meet demand while minimizing costs.

**Cutting stock** (for lumber, paper, etc.) involves allocating space on large sheets or timbers to be cut into smaller pieces, to meet demand while minimizing waste.

3. Distribution and Networks

**Routing** (of goods, natural gas, electricity, digital data, etc.) involves allocating something to different paths through which it can move to various destinations, to minimize costs or maximize throughput.

**Loading** (of trucks, rail cars, etc.) involves allocating space in vehicles to items of different sizes so as to minimize wasted or unused space.

**Scheduling** of everything from workers to vehicles and meeting rooms involves allocating capacity to various tasks in order to meet demand while minimizing overall costs.

Spreadsheets such as Microsoft Excel provide a convenient way to build such models. Cells on a worksheet can hold numbers, labels, or formulas that calculate new values:

1. <u>The decision variables</u> for a model are simply worksheet cells containing numbers that Solver can change.

2. <u>The objective</u> is a cell containing a formula you want Solver to maximize (or minimize) by adjusting the values of the decision variable cells.

3. <u>Constraints</u> are logical conditions on formula cells that must be satisfied (specified with  $\leq =$ , = or  $\geq =$  relations). Frontline's various Solver products provide powerful tools for solving, or optimizing, such models.

# **1. Decision Variables**

Start with the decision variables. They usually measure the amounts of resources used (such as money to be allocated to some purpose) or the level of various activities to be performed (such as the number of products to be manufactured, the number of pounds or gallons of a chemical to be blended). For example, if you are shipping goods from 3 different plants to 5 different warehouses (as shown below) there are  $3 \times 5 = 15$  different routes along which products could be shipped. So, you would likely have 15 decision variables, each one representing the amount of product shipped from a particular plant to a particular warehouse.



Now, suppose each plant produces 4 different product types and you want to determine the optimal shipping plan in each of the next 6 months. This might lead to  $3 \times 4 \times 5 \times 6 = 360$  decision variables. This simple example illustrates how a model can become large rather quickly. So part of the art of modeling is deciding how much detail is really required in a given situation.

# 2. Objective Function

Once you've defined the decision variables, the next step is to define the objective, which is normally some function that depends on the decision variables. As a simple example, suppose you were planning how many units to manufacture of three different types of TV sets: Plasma, LCD, and LED.

Your objective might be to maximize profit, so assume that each Plasma TV yields a profit of \$75, each LCD \$50, and each LED \$35. Then your objective (computing profit) could be expressed as:

75 \* Number of Plasma sets + 50 \* Number of LCD sets + 35 \* Number of LED sets

On a spreadsheet where the number of Plasma, LCD and LED TV sets are in cells D9, E9 and F9, respectively, the formula for the objective cell could be:

= 75\*D9 + 50\*E9 + 35\*F9

However, the values 75, 50, and 35 (known as the objective function coefficients) should probably be entered in individual cells in the spreadsheet, say, cells D8, E8, and F8, respectively. In that case, the formula for the objective would be:

= D8\*D9 + E8\*E9 + F8\*F9

In some cases, an optimization model is defined completely by its decision variables and objective. However, most optimizations problems involve constraints that restrict the values the decision variables can assume and, in turn, the objective value that can be achieved.

### **3. Defining Constraints**

Constraints are logical conditions that a solution to an optimization problem must satisfy. They reflect real-world limits on production capacity, market demand, available funds, and so on. To define a constraint, you first compute the value of interest using the decision variables. Then you place an appropriate limit (<=, = or >=) on this computed value. The following examples illustrate a variety of types of constraints that commonly occur in optimization problems.

a) General Constraints

Suppose that cells A1:A5 contain the percentage of funds to be invested in each of 5 stocks. We would want the sum of these cells to equal 1 (or 100%). To accomplish this, in cell B1 you might calculate the sum of the percentages as =SUM(A1:A5) and then use solver to define a constraint to require that cell B1 = 1.

As another example, suppose a company has an advertising budget of \$50,000 for the coming month and TV and newspaper ads cost \$3,000 and \$500 per ad, respectively. If cells C3 and D3 represent decision variables for, respectively, the number of TV ads and the number of newspaper ads purchased, we could calculate the total amount spent of advertising in, say, cell E3 as =3000\*C3 + 500\*D3. We would then use solver to define a constraint requiring that E3<=50000.

# b) Bounds on Variables

You can also place a constraint directly on a decision variable, such as A1  $\leq$  100 or B7  $\geq$  5. These types of upper and lower bounds on the variables are handled efficiently by most optimizers and are very useful in many problems. For example, if your decision variables measure the number of products of different types that you plan to manufacture, producing a negative number of products would make no sense. This type of non-negativity constraint is very common. Although it may be obvious to you, non-negativity constraints such as A1  $\geq$  0 must be communicated to the solver so that it knows that negative values are not allowed.

c) Policy Constraints

Some constraints are determined by policies that you or your organization may set. For example, in a portfolio optimization, you might have a limit on the maximum percentage of funds to be invested in any one stock, or one industry group.

# d) Physical Constraints

Many constraints are determined by the physical nature of the problem. For example, suppose you are modeling product shipments in and out of a warehouse over time. You'll probably need a balance constraint to specify that, in each time period, the beginning inventory plus the products received minus the products shipped out equals the ending inventory. And, of course, the ending inventory in one period becomes the beginning inventory for the next period.

# e) Integer Constraints

Optimization software also allows you to specify constraints requiring decision variables to assume only integer (whole number) values in the final solution. For example, if you are scheduling a fleet of trucks, a solution that calls for a fraction of a truck to travel a certain route would not be useful. Integer constraints normally can be applied only to decision variables, not to quantities calculated from them.

A special type of integer constraint specifies that a variable must be binary -- either 0 or 1 -- at the final solution. Binary variables can be used to model "yes/no" or "go/no-go" decisions and are very useful in a variety of modeling situations. For example, you might use a 0-1 or binary integer variable to represent a decision about whether to lease a new machine. Your model could use this binary variable to include (or exclude) the monthly fixed lease cost for the machine in the objective as well as to create a lower cost per item processed with the machine, if it is used. In this way, a solver can determine whether or not the machine should be leased. Some problems involve determining an optimal ordering of items. For example, we might want to determine the optimal ordering of jobs on a machine where set-up costs on the machine vary depending on the order of the jobs. As another example, we might want to determine the optimal order in which to deliver packages to customers throughout a city while minimizing total travel distance. In these situations, solver offers a special type of integer constraint (known as an "alldifferent" constraint) where the values of n decision variables must be a permutation of integers from 1 to n.

# Step by Step - Product Mix Example 1.

# The Example Problem

Imagine that you manage a factory which produces **four different types** of wood paneling. Each type of paneling is made by gluing and pressing together a different mixture of pine and oak chips. The following table summarizes the required amount of gluing, pressing, and mixture of wood chips required to produce a pallet of 50 units of each type of paneling:

|                     | Resour | ces Required | Per Pallet of Paneli | ng Type |
|---------------------|--------|--------------|----------------------|---------|
|                     | Tahoe  | Pacific      | Savannah             | Aspen   |
| Glue (Quarts)       | 50     | 50           | 100                  | 50      |
| Pressing (Hours)    | 5      | 15           | 10                   | 5       |
| Pine chips (Pounds) | 500    | 400          | 300                  | 200     |
| Oak Chips (Pounds)  | 500    | 750          | 250                  | 500     |

Let's assume that for the next production cycle, you have 5,800 quarts of glue; 730 hours of pressing capacity; 29,200 pounds of pine chips; and 60,500 pounds of oak chips available. Further, assume that each pallet of Tahoe, Pacific, Savannah, and Aspen panels can be sold for profits of \$450, \$1,150, \$800, and \$400, respectively.

## Writing the Formulas

Before we implement this problem statement in either Excel or Visual Basic, let's write out formulas corresponding to the verbal description above. If we temporarily use the symbol X1 for the number of Tahoe pallets produced, X2 for the number of Pacific pallets produced, and X3 for the number of Savannah pallets produced, and X4 for the number of Aspen pallets produced, the objective (calculating total profit) is:

Maximize 450 X1 + 1150 X2 + 800 X3 + 400 X4

A pallet of each type of panel requires a certain amount of glue, pressing, pine chips, and oak chips. The amount of resources used (calculated by the left hand side of each constraint) depends on the mix of products built, and we have a limited amount of each type of resource available (corresponding to the constraint right hand side values). The constraints for this problem are expressed as follows:

Subject to: (These rules are entered into the solver one at a time)

50 X1 + 50 X2 + 100 X3 + 50 X4 <= 5,800 (Glue) 5 X1 + 15 X2 + 10 X3 + 5 X4 <= 730 (Pressing) 500 X1 + 400 X2 + 300 X3 + 200 X4 <= 29,200 (Pine Chips) 500 X1 + 750 X2 + 250 X3 + 500 X4 <= 60,500 (Oak Chips)

Since the number of products built cannot be negative, we'll also have non-negativity conditions on the variables:

X1, X2, X3, X4 >= 0

Now, we'll take you step by step through implementing and solving this optimization model using Excel's built-in Solver.

## **The Essential Steps**

- 1. To define an optimization model in Excel you'll follow these essential steps:
- 2. Organize the data for your problem in the spreadsheet in a logical manner.
- 3. Choose a spreadsheet cell to hold the value of each decision variable in your model.
- 4. Create a spreadsheet formula in a cell that calculates the objective function for your model.
- 5. Create formulas in cells to calculate the left hand sides of each constraint.

Use the dialogs in Excel to tell Solver about your decision variables, the objective, contraints, and desired bounds on constraints and variables.

## Run Solver to find the optimal solution.

Within this overall structure, you have a great deal of flexibility in how you choose cells to hold your model's decision variables and constraints, and which formulas and built-in functions you use. In general, your goal should be to create a spreadsheet that communicates its purpose in a clear and understandable manner.

# Creating an Excel Worksheet

You should take a copy of the workbook used for this example from your teacher and use it as a guide to accomplish your own example.

Assuming that you have organized the data for the problem in Excel, the next step is to create a worksheet where the formulas for the objective function and the constraints are calculated. Because decision variables and constraints usually come in logical groups, you'll often want to use cell ranges in your spreadsheet to represent them.

In the worksheet below, we have reserved cells B4, C4, D4 and E4 to represent our decision variables X1, X2, X3, and X4 representing the number of pallets of each type of panel to produce. The Solver will determine the optimal values for these cells.

|    | L10        | • (      | <b>a</b> | fx            |         |                     |           |        |    |   |   |
|----|------------|----------|----------|---------------|---------|---------------------|-----------|--------|----|---|---|
| 14 | A          | В        | С        | D             | E       | F                   | G         | н      | L. | L | ĸ |
| 1  |            |          |          |               |         |                     |           |        |    |   | 1 |
| 2  |            |          | Pane     | el Type       |         |                     |           |        |    |   |   |
| 3  |            | Tahoe    | Pacific  | Savannah      | Aspen   | -                   |           |        |    |   |   |
| 4  | Pallets    | 0        | 0        | 0             | 0       | <b>Total Profit</b> |           |        |    |   |   |
| 5  | Profit     | \$450    | \$1,150  | \$800         | \$400   | \$0                 | 4         |        |    |   |   |
| 6  |            |          |          |               |         |                     | -         |        |    |   |   |
| 7  |            | Resource | es Requi | red per Palle | et Type | Used                | Available |        |    |   |   |
| 8  | Glue       | 50       | 50       | 100           | 50      | 0                   | 5,800     | quarts |    |   |   |
| 9  | Pressing   | 5        | 15       | 10            | 5       | 0                   | 730       | hours  |    |   |   |
| 10 | Pine Chips | 500      | 400      | 300           | 200     | 0                   | 29,200    | pounds |    |   |   |
| 11 | Oak Chips  | 500      | 750      | 250           | 500     | 0                   | 60,500    | pounds |    |   |   |
| 12 |            |          |          |               |         |                     |           |        |    |   |   |

Notice that the profit for each pallet of panels (\$450, \$1,150, \$800 and \$400) was entered in cells B5, C5, D5 and E5, respectively. This allows us to compute the objective in cell F5 as:

Formula for cell F5: =B5\*B4+C5\*C4+D5\*D4+E5\*E4

or, equivalently,

Formula for cell F5: =SUMPRODUCT(B5:E5,B4:E4)

In cells B8:E11, we've entered the amount of resources needed to produce a pallet of each type of panel. For example, the value 15 in cell C9 means that 15 hours of pressing is required to produce a pallet of Pacific style panels. These numbers come directly from the formulas for the constraints shown earlier. With these values in place, we can enter a formula in cell F8 to compute the total amount of glue used for any number of pallets produced:

Formula for cell F8: =SUMPRODUCT(B8:E8,\$B\$4:\$E\$4)

We can copy this formula to cells F9:F11 to compute the total amount of pressing, pine chips, and oaks chips used. (The dollar signs in \$B\$4:\$E\$4 specify that this cell range stays constant, while the cell range B8:E8 becomes B9:E9, B10:E10, and B11:E11 in the copied formulas.) The formulas in cells F8:F11 correspond to the left hand side values of the constraints.

In cells G8:G11, we've entered the available amount of each type of resource (corresponding to the right hand side values of the constraints). This allows us to express the constraints shown earlier as:

F8:F11<=G8:G11

This is equivalent to the four constraints:  $F8 \le G8$ ,  $F9 \le G9$ ,  $F10 \le G10$ , and  $F11 \le G11$ . We can enter this set of constraints directly in the Solver dialogs along with the non-negativity conditions:

D9:F9 >= 0

## Using Excel's built-in Solver Dialogs

To let the Solver know which cells on the worksheet represent the decision variables, constraints and objective function, we click the Solver button on the Data tab, or the Premium Solver button on the Add-Ins tab, which displays the Solver Parameters dialog. In the Set Objective (or Set Target Cell) edit box, we type or click on cell F5, the objective function. In the By Changing Variable Cells edit box, we type B4:E4 or select these cells with the mouse.

|              |        | e //      | f.           |         |              |           |        |   | 1 1000 |
|--------------|--------|-----------|--------------|---------|--------------|-----------|--------|---|--------|
| A            | - D    | ¢         | 0            | E.      | F.           | 6         | H      | Set Objective: grss   | 1      |
| 2            |        | Pane      | Type         |         |              |           |        | The state of the state of the   |        |
| 1            | Tahoe  | Pacific   | Savannah     | Aspen   |              |           |        | w a liter O wild O Rame out   | _      |
| Pallets      | 0      | 0         | 0            | 0       | Total Profit |           |        | By Changing Variable Cels:  |        |
| Profit       | \$450  | \$1,150   | \$800        | \$400   | \$0          | 1         | -      | \$8\$4:\$E14  | 16     |
| Ē            |        |           |              |         |              | 110000000 |        |   |        |
| 7.           | Resour | ces Requi | red per Pall | et Type | Used         | Available |        | Signed to the constraints:  | -1 I   |
| Glue         | 50     | 50        | 100          | 50      | 0            | 5,800     | quarts |   | 49     |
| Pressing     | 5      | 15        | 10           | 5       | 0            | 730       | hours  | Change  |        |
| 0 Pine Chips | 500    | 400       | 300          | 200     | 0            | 29,200    | pounds |   | 201    |
| 1 Oak Chips  | 500    | 750       | 250          | 500     | 0            | 60,500    | pounds | Qelete  |        |
| 2            |        |           |              |         |              |           | -      | The second se |        |
| 3            |        |           |              |         |              |           |        | Beset Al  |        |
| 5            |        |           |              |         |              |           | -      | - Load/Save   |        |
| 6            |        |           |              |         |              |           |        | V Make Unconstrained Variables Non-Negative   | -      |
| 7            |        |           |              |         |              |           |        | Education and the second  | -12    |
| 8            |        |           |              |         |              |           |        | spect a saving vertice. Singlex UP Options  |        |
| 9            |        |           |              |         |              |           |        | Solving Method  |        |
| 0            |        |           |              |         |              |           |        | Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simples            | ĸ      |
| 1            |        |           |              |         |              |           |        | engine for linear solver moderns, and seect the Evolutionary engine for solver problems that are<br>non-smooth. |        |
| 2            |        |           |              |         |              |           |        | 199789762   |        |
| 4            |        |           |              |         |              |           | -      |   |        |
| 5            |        |           |              |         |              |           |        | Help Solve Ogse   | 10     |
| 6            |        |           |              |         |              |           | 1      |   | _      |

To add the constraints, we click on the Add button in the Solver Parameters dialog and select cells F8:F11 in the Cell Reference edit box (the left hand side), and select cells G8:G11 in the Constraint edit box (the right hand side); the default relation  $\leq$  is OK.

| Cell Reference: |    | Constraint:     |          |
|-----------------|----|-----------------|----------|
| \$F\$8:\$F\$11  | <= | =\$G\$8:\$G\$11 | <b>1</b> |

We choose the Add button again (either from the Add Constraint dialog above, or from the main Solver Parameters dialog) to define the non-negativity constraint on the decision variables. (Alternatively, we can check the Make Unconstrained Variables Non-Negative option in the Solver Parameters dialog.)

When we've completely entered the problem, the Solver Parameters dialog appears as shown below. This is the Solver dialog from Excel 2010; the Solver in earlier versions of Excel have similar

elements. Frontline's Premium Solver products can emulate either style, and they also offer a new Ribbon-based user interface.

| Set Objective:                                      | \$F\$5                           |   |  |   |
|---|----------------------------------|---|--|---|
| To: 💿 Max   | © Mi <u>n</u>                    | O Value Of:                                       | 0  |   |
| By Changing Variable Cel                            | ls:                              |   |  | (62   |
| \$8\$4:\$E\$4                                       |                                  |   |  | LE2   |
| Subject to the Constraint                           | is:                              |   |  |   |
| \$8\$4:\$E\$4 >= 0<br>\$F\$8:\$F\$11 <= \$G\$8:\$C  | 5\$11                            |   | *  | Add   |
|   |                                  |   |  | Change  |
|   |                                  |   |  | Delete  |
|   |                                  |   |  | Reset All   |
|   |                                  |   | -  | Load/Save   |
| Make Unconstrained                                  | Variables Non                    | -Negative   |  | ·   |
| Select a Solving Method:                            | S                                | mplex LP  | •  | Options   |
| Solving Method                                      |                                  |   |  |   |
| Select the GRG Nonlinea<br>engine for linear Solver | ar engine for S<br>Problems, and | Solver Problems that a<br>d select the Evolutiona | re smooth nonline<br>ary engine for Solv | ar. Select the LP Simplex<br>er problems that are |

# Finding and Using the Solution

To find the optimal solution, we simply click on the Solve button. After a moment, the Solver returns the optimal solution in cells B4 through E4. This means that we should build 23 pallets of Tahoe panels, 15 pallets of Pacific panels, 39 pallets of Savannah panels, and 0 pallets of Aspen panels. This results in a total profit of \$58,800 (shown in cell F5).

| X  | 3 - 6      | •   •  |           |              |         |                     |           | PanelManuf | facturing - Microsoft Excel  |     |
|----|------------|--------|-----------|--------------|---------|---------------------|-----------|------------|--|-----|
| 1  | ile Home   | Insert | Page L    | ayout For    | mulas   | Data Revie          | w View    | Develo     | Solver Results   |     |
|    | F5         | • (    | ÷         | fx =SUMP     | RODUCT  | (B5:E5, B4:E4)      |           |            |  | 1   |
| 1  | A          | В      | С         | D            | E       | F                   | G         | н          | Solver found a solution. All Constraints and optimality<br>conditions are satisfied. Reports |     |
| 2  |            |        | Pane      | Type         |         |                     |           |            | Answer   |     |
| 3  |            | Tahoe  | Pacific   | Savannah     | Aspen   | in the              |           |            | Umits  |     |
| 4  | Pallets    | 23     | 15        | 39           | 0       | <b>Total Profit</b> |           |            | O Bestore Original Values  |     |
| 5  | Profit     | \$450  | \$1,150   | \$800        | \$400   | \$58,800            |           |            |  |     |
| 6  |            |        |           |              |         |                     | -         |            | Return to Solver Parameters Dialog Outline Reports   |     |
| 7  |            | Resour | ces Requi | red per Pall | et Type | Used                | Available |            |  | å I |
| 8  | Glue       | 50     | 50        | 100          | 50      | 5800                | 5,800     | quarts     | QK Cancel Save Scenario  | 11  |
| 9  | Pressing   | 5      | 15        | 10           | 5       | 730                 | 730       | hours      |  |     |
| 10 | Pine Chips | 500    | 400       | 300          | 200     | 29200               | 29,200    | pounds     | Solver found a solution. All Constraints and optimality conditions are                       |     |
| 11 | Oak Chips  | 500    | 750       | 250          | 500     | 32500               | 60,500    | pounds     | satisfied.   | Ш   |
| 12 |            |        |           |              |         |                     |           |            | solution. When Simplex LP is used, this means Solver has found a global                      | Ш   |
| 13 |            |        |           |              |         |                     |           |            | optimal solution.  |     |
| 14 |            |        |           |              |         |                     |           |            |  |     |

The message "Solver found a solution" appears in the Solver Results dialog, as shown above. We now click on "Answer" in the Reports list box to produce an Answer Report, and click OK to keep the optimal solution values in cells B4:E4.

After a moment, the Solver creates another worksheet containing an Answer Report, like the one below, and inserts it on a new tab to the left of the problem worksheet in the Excel workbook ("Answer Report 1").

| K  |          | + (8    | ~ 🖵        |         |            | P        | anelMan    | ufactu | uring - M  | licros  | oft Excel |            |        |           | (inc |     | ×         |   |
|----|----------|---------|------------|---------|------------|----------|------------|--------|------------|---------|-----------|------------|--------|-----------|------|-----|-----------|---|
|    | File     | Home    | Insert     | Page    | Layout     | Formu    | las Data   | Re     | view Vie   | w D     | eveloper  | Add-Ins    | Risk   | Solver Pl | v    | 0 - | - #       | x |
|    |          | A1      |            | • (6    |            | fx       | Microso    | ft Exe | cei 14.0   | Answ    | er Repo   | ort        |        |           |      |     |           | 0 |
|    | A B      |         | с          |         | D          |          | E          |        | F          |         | G         | н          | 1      | L         | 1    | K   | L         | E |
| 1  | Microso  | ft Exce | 1 14.0 Ans | wer R   | eport      |          |            |        |            |         |           |            |        |           |      |     | 1         | F |
| 2  | Worksh   | eet: [P | anelManut  | facturi | ng.xls]Sh  | eet1     |            |        |            |         |           |            |        |           |      |     |           |   |
| 3  | Report   | Created | 1: 9/14/20 | 11 3:1  | 6:31 PM    |          |            |        |            |         | 192022100 |            |        |           |      |     |           |   |
| 4  | Result:  | Solver  | found a so | lution  | . All Cons | straints | and optin  | nality | conditions | s are s | atisfied. |            |        |           |      |     |           |   |
| 5  | Solver E | ingine  | 0.00022    |         |            |          |            |        |            |         |           |            |        |           |      |     |           |   |
| 6  | Engi     | ne: Sin | nplex LP   |         | 2222       |          |            |        |            |         |           |            |        |           |      |     |           |   |
| 7  | Solu     | tion Ti | me: 0.016  | Seco    | nds.       |          |            |        |            |         |           |            |        |           |      |     |           | l |
| 8  | Itera    | itions: | 3 Subpro   | blem:   | s: 0       |          |            |        |            |         |           |            |        |           |      |     |           |   |
| 3  | Solver   | Jptions | Intimizer  |         |            | to Has I | and Breek  |        | 0.000001   |         |           |            |        |           |      |     |           |   |
| 10 | Max      | Subas   | phimited   | , iter  | ations u   | Internet | ted, Preci | sion   | 0.000001   |         | lamara 1  | 6/ Acres   | na Nas | Monstine  | 5    |     |           |   |
| 12 | Max      | Suppr   | obients u  | miimi   | teo, max   | integ    | er sols u  | niimi  | teo, integ | ger to  | rerance 1 | .76, ASSUI | ne Non | ivegative |      |     |           |   |
| 12 |          |         |            |         |            |          |            |        |            |         |           |            |        |           |      |     |           |   |
| 14 | Ohierti  |         | (May)      |         |            |          |            |        |            |         |           |            |        |           |      |     |           |   |
| 15 | Cell     | ve çen  | Name       |         | Original   | Value    | Einal Va   | lue    | -          |         |           |            |        |           |      |     |           |   |
| 15 | CECE     | Drof    | Total P    | ofit    | Unginar    | co       | Filidi Vo  | 2 900  |            |         |           |            |        |           |      |     |           |   |
| 17 | 3135     | 101     | it fotal P | Unt     |            | 30       | 330        | 5,000  | -          |         |           |            |        |           |      |     |           |   |
| 18 |          |         |            |         |            |          |            |        |            |         |           |            |        |           |      |     |           |   |
| 19 | Variah   | e Cell  | <          |         |            |          |            |        |            |         |           |            |        |           |      |     |           | 1 |
| 20 | Cel      | ie een  | Name       |         | Original   | Value    | Einal Va   | atue   | Intege     | er.     |           |            |        |           |      |     |           |   |
| 21 | CRCA     | Pall    | etc Tabou  |         | originor   | 0.00     | T HIGH VC  | 23     | Contin     | -       |           |            |        |           |      |     |           |   |
| 22 | SCSA     | Pall    | etc Parifi | r       |            | 0        |            | 15     | Contin     | -       |           |            |        |           |      |     |           |   |
| 23 | SDS4     | Pall    | ets Savar  | nah     |            | 0        |            | 30     | Contin     |         |           |            |        |           |      |     |           |   |
| 74 | SESA     | Pall    | ets Asner  | 0       |            | 0        | 1          | 0      | Contin     |         |           |            |        |           |      |     |           |   |
| 25 |          | 1911    | ets roper  |         |            |          |            |        | contait    |         |           |            |        |           |      |     |           |   |
| 26 |          |         |            |         |            |          |            |        |            |         |           |            |        |           |      |     |           |   |
| 27 | Constra  | sints   |            |         |            |          |            |        |            |         |           |            |        |           |      |     |           |   |
| 28 | Cell     |         | Name       |         | Cell Va    | lue      | Formu      | la     | Status     | s s     | lack      |            |        |           |      |     |           |   |
| 29 | SECR     | Glue    | Used       | _       | CCI II     | 5800     | SESR<=SO   | :58    | Binding    |         | 0         |            |        |           |      |     |           |   |
| 30 | SESO     | Pres    | singlise   | d       |            | 730      | SES9<=50   | :50    | Binding    |         | 0         |            |        |           |      |     |           |   |
| 31 | SES1     | 0 Pine  | Chins U    | sed     | 1          | 29200    | SES104=S   | G\$10  | Binding    |         | 0         |            |        |           |      |     |           |   |
| 32 | SES1     | 1 Oak   | Chins Us   | ed      |            | 32500    | SES11<=S   | G\$11  | Not Bind   | ting 2  | 8000      |            |        |           |      |     |           |   |
| 33 | SBS4     | Pall    | ets Tahor  |         |            | 23       | SBS4>=0    |        | Not Bind   | ding    | 23        |            |        |           |      |     |           |   |
| 34 | SCS4     | Pall    | ets Pacifi | c       |            | 15       | SCS4>=0    |        | Not Bind   | ding    | 15        |            |        |           |      |     |           |   |
| 35 | SDS4     | Pall    | ets Savar  | nah     |            | 39       | SD\$4>=0   |        | Not Bind   | ding    | 39        |            |        |           |      |     |           | L |
| 36 | SES4     | Pall    | ets Asper  | n       |            | 0        | \$E\$4>=0  |        | Binding    |         | 0         |            |        |           |      |     |           |   |
| 37 | -        |         |            |         |            |          |            |        |            |         |           |            |        |           |      |     |           | 1 |
| М  | 4 1 1    | Ans     | wer Rep    | ort 1   | Prob       | lem ,    | 12/        |        |            |         |           |            | 110    | 1         |      |     |           | Ĺ |
| Re | ady P    |         |            |         |            |          |            |        |            |         |           |            | 85%    | 0         | -0   | 1   | $ \oplus$ |   |

This report shows the original and final values of the objective function and the decision variables, as well as the status of each constraint at the optimal solution. Notice that the constraints on glue, pressing, and pine chips are binding and have a slack value of 0. The optimal solution would use up all of these resources; however, there were 28,000 pounds of oak chips left over. If we could obtain additional glue, pressing capacity, or pine chips we could further increase total profits, but extra oak chips would not help in the short run

**Assignment 1.** Practice on your own the example presented above and describe the results in the report for this practical training.

|    | A                            | В           | С            | D      | E         |
|----|------------------------------|-------------|--------------|--------|-----------|
| 1  | Maximisat                    | ion of Cont | tribution Ma | rgin   |           |
| 2  |                              |             |              |        |           |
| 3  |                              | Goo         | ds           |        |           |
| 4  |                              | A           | в            |        |           |
| 5  | Revenue per unit             | SFr. 1'000  | SFr. 3'000   |        |           |
| 6  | Direct Cost per unit         | SFr. 700    | SFr. 2'500   |        |           |
| 7  | Contribution Margin per unit | SFr. 300    | SFr. 500     |        |           |
| 8  |                              |             |              |        |           |
| 9  |                              |             |              | Capaci | ties      |
| 10 | Production time in [h/ME]    |             |              | used   | available |
| 11 | Machine I                    | 1           | 2            | 0      | 170       |
| 12 | Machine II                   | 1           | 1            | 0      | 150       |
| 13 | Machine III                  | 0           | 3            | 0      | 180       |
| 14 |                              |             |              |        |           |
| 15 | Units of Goods Produced      | 0           | 0            |        |           |
| 16 | Contribution Margin          | SFr. 0      | SFr. 0       |        |           |
| 17 |                              | -           |              |        |           |
| 18 | Total Contribution Margin    |             |              | SFr. 0 |           |
| 19 |                              |             |              |        |           |

#### Example:

A firm produces two goods A and B. Each good yields different revenues (B5,C5) and causes certain direct costs (B6,C6). By computing the difference between revenue and direct cost, we get the *contribution margin*. (B7,C7). The production of both goods requires the use of three machines I, II and III. In order to produce 1 unit of good A, we need 1 hour of machine I and 1 hour of machine II (B11-B13). Production of good B, however, is more involved. In order to produce 1 unit of B, it takes 2 hours on machine I, 1 hour on machine II and 3 hours on machine III (C11-C13). When producing the goods, we have to take into account that each machine can only be run for a limited number of hours. The total capacity of machines I, II and III are respectively 170, 150 and 180 hours (E11-E13).

The problem is to find the number of units of goods produced (for both goods), such that the total contribution margin is maximised subject to the capacity constraints.

To solve this problem, we have to set up an **objective function** and several **constraints**. The objective function has to be specified within the Excel worksheet, while the constraints need merely be stated in the Solver window.

|    | A                            | В           | С            | D      | E         |
|----|------------------------------|-------------|--------------|--------|-----------|
| 1  | Maximisat                    | ion of Cont | tribution Ma | rgin   |           |
| 2  |                              |             |              |        |           |
| 3  |                              | Goo         | ods          |        |           |
| 4  |                              | Α           | В            |        |           |
| 5  | Revenue per unit             | SFr. 1'000  | SFr. 3'000   |        |           |
| 6  | Direct Cost per unit         | SFr. 700    | SFr. 2'500   |        |           |
| 7  | Contribution Margin per unit | SFr. 300    | SFr. 500     |        |           |
| 8  |                              |             |              |        |           |
| 9  |                              |             |              | Capaci | ties      |
| 10 | Production time in [h/ME]    |             |              | used   | available |
| 11 | Machine I                    | 1           | 2            | 0      | 170       |
| 12 | Machine II                   | 1           | 1            | 0      | 150       |
| 13 | Machine III                  | 0           | 3            | 0      | 180       |
| 14 |                              |             |              |        |           |
| 15 | Units of Goods Produced      | 0           | 0            |        |           |
| 16 | Contribution Margin          | SFr. 0      | SFr. 0       |        |           |
| 17 |                              |             |              |        |           |
| 18 | Total Contribution Margin    |             |              | SFr. 0 |           |
| 19 |                              |             |              |        |           |

#### **Objective Function**

In order to figure out what the total contribution margin is, we first of all have to find the contribution margins per unit for each good.

The **contribution margin per unit** is the difference between the revenue per unit and the cost per unit:

- Good A: Cell B7: = B5-B6
- Good B: Cell C7: = C5-C6

The total contribution margin of each good is then found by multiplying the contribution margin per unit with the number of units of goods produced:

- Good A: Cell B16: = B7\*B15
- Good B: Cell C16: = C7\*C15

Finally, the **total contribution margin** is the sum of the total contribution margins of goods A and B:

• Total contribution margin: Cell D18: = B16+C16

#### Constraints

There are three constraints involved in the present maximisation problem, reflecting the fact that each of the three machines' capacity limit must not be exceeded. However, before we are able to specify these constraints in the Solver, we need to take some preliminary steps. In particular, we have to find and state formulas which compute the number of machine hours used in production of good A and B. For each machine, this can be computed by multiplying the production times with the number of units of goods produced:

- Used capacity of machine I: Cell D11: = B11\*\$B\$15+C11\*\$C\$15
- Used capacity of machine II: Cell D12: = B12\*\$B\$15+C12\*\$C\$15
- Used capacity of machine III: Cell D13: = B13\*\$B\$15+C13\*\$C\$15

These formulas are put in cells D11-D13. Having done so, the three constraints can be specified as follows:

- D11 ≤ E11
- D12 ≤ E12
- D13 ≤ E13





2 X Solver Parameters Set Target Cell: 5D\$18 Solve ( Max CMin Equal To: 0 C Value of: Close By Changing Cells \$8\$15:\$C\$15 ٩. Guess Subject to the Constraints: Options \$D\$11:\$D\$13 <= \$E\$11:\$E\$13 -Add Change Reset All Delete Help



#### Answer report:

This report lists the values of the target cell and variable cells before and after the solution. Moreover, we are given further information on the constraints.

#### Sensitivity report:

This report provides information on how sensitive the solution is to small changes in the constraints or in the formula of the target cell.

Having specified the problem, we can now solve it with the aid of Solver.

Solver is found in the menu "Tools". After a click on "Solver" the dialogue box Solver Parameters pops up.

In analogy to the Goal Seek tool, we firstly have to define the **target cell** and what this target cell should be **equal to**. There are three options available:

- If Max is chosen, the value of the target cell will be maximised.
- If Min is chosen, the value of the target cell will be minimised.
- By choosing Value of, we can determine the value that the target cell should attain.

As a next step, we have to indicate which cells can be varied in order to reach the target value. This can be done in the section **By Changing Cells**. In contrast to Goal Seek, more than one variable cell can be specified here.

Eventually, we have to define the constraints to our optimisation problem. By clicking on "Add" we reach the dialogue box Add Constraint, where we can specify the restrictions. We firstly have to state in Cell Reference, which cells are required to meet the restriction. In Constraint we then have to enter the cells which contain the restriction. Moreover, an operator has to be selected in order to characterise the constraint. A click on "OK" concludes the definition of the restriction. Further constraints can be added by repeating the procedure just described.

Let us now use the Solver tool to compute a solution to our above problem of maximising the total contribution margin.

For this purpose, we have to enter all the necessary specifications in the dialogue box **Solver Parameters**. Before we start the solution procedure (by clicking on "Solve") the dialogue box might look as follows:

- Target Cell:
- Equal to: Max
- Changing Cells: \$B\$15:\$C\$15
- Constraints: SD\$11:SD\$13<=SE\$11:SE\$13</li>

**SDS18** 

Note: In order to avoid negative numbers in the result, it is sometimes advisable to introduce non-negativity constraints.

After the solution of the problem, the window **Solver Results** pops up. It contains brief information on whether the computations led to a solution or not. In the present case a solution that satisfies all constraints and optimality conditions was found.

In addition to that, Excel allows you to save the solution as a scenario and provides you with three reports:

#### Limits report:

This report lists the target cell and the variable cells together with their actual values, upper and lower limits and target values.

|    | A                            | В           | С           | D           | E         |
|----|------------------------------|-------------|-------------|-------------|-----------|
| 1  | Maximisa                     | tion of Cor | tribution M | argin       |           |
| 2  |                              |             |             |             |           |
| 3  |                              | Go          | ods         |             |           |
| 4  |                              | A           | В           |             |           |
| 5  | Revenue per unit             | SFr. 1'000  | SFr. 3'000  |             |           |
| 6  | Direct Cost per unit         | SFr. 700    | SFr. 2'500  |             |           |
| 7  | Contribution Margin per unit | SFr. 300    | SFr. 500    |             |           |
| 8  | 1994 A                       |             |             |             |           |
| 9  |                              |             |             | Capaci      | ties      |
| 10 | Production time in [h/ME]    |             |             | used        | available |
| 11 | Machine I                    | 1           | 2           | 170         | 170       |
| 12 | Machine II                   | 1           | 1           | 150         | 150       |
| 13 | Machine III                  | 0           | 3           | 60          | 180       |
| 14 |                              |             |             |             |           |
| 15 | Units of Goods Produced      | 130         | 20          |             |           |
| 16 | Contribution Margin          | SFr. 39'000 | SFr. 10'000 |             |           |
| 17 |                              |             |             |             |           |
| 18 | Total Contribution Margin    |             |             | SFr. 49'000 |           |
| 19 |                              |             |             |             |           |

Having clicked "OK" in the **Solver Results** window, the results of the Solver are transferred to the current worksheet. Accordingly, the total contribution margin is maximised if we produce

- 130 units of good A and
- 20 units of good B.

Thereby the capacities of machines I and II are fully exploited, whereas only a third of the capacity of machine III is used (D11 - D13).

The two goods yield a contribution margin of respectively

- Product A: SFr. 39'000 (Cell B16)
- Product B: SFr. 10'000 (Cell C16)

On the whole, the total contribution margin realised through this production plan is equal to SFr. 49'000 (Cell D18).

**Assignment 2.** Practice on your own the example presented above and describe the results in the report for this practical training.

# !!! PREPARE A REPORT CONTAINING THE RESULT FOR ALL ASSIGNMENTS WITH ACCOMPANYING EXPLANATIONS